Binomial Formula

P(r,n,p) = $\frac{n(n-1)\,...\,(n-r+1)}{r(r-1)\,...\,1}\; p^r(1-p)^{n-r}$ is the probability of getting exactly r successes in n trials when the probability of a success in one trial is p.

Derivation:

The probability of getting r successes in particular positions among the n trials and n-r failures in the remaining positions is: $p^r(1-p)^{n-r}$.
This must be multiplied by the number of different positions in which the successes can occur.

Let’s calculate the number of ways of selecting r distinct numbers from n distinct numbers.
There are n numbers that can be chosen first. This leaves n-1 numbers that can be chosen second. This leaves n-2 that can be chosen third and so on. Since we are only choosing r numbers, we will have n-r+1 numbers that can be chosen last. So the total number of ways of selecting r distinct numbers ftom n distinct numbers is n(n-1) … (n-r+1).

Example: Let n = 3 and let the n numbers be 1,2,3.
Let r = 2, then the possible selections are 1 and 2, 2 and 1, 1 and 3, 3 and 1, 2 and 3, and 3 and 2. This is a total of 6 possible selections. N-r+1 = 3-2+1 = 2. So using the formula above we get 3 x 2 = 6 ways which agrees with the number of listed selections. Notice that the same two numbers get counted twice in this selection. In general, this selection will count a specific r numbers in all possible ways that the r numbers can be arranged.

Let's calculate the number of ways of arranging r distinct numbers.
There are r different numbers that can be chosen for the first position. This leaves r-1 numbers which can be chosen for the second position. This leaves r-2 numbers for the third position and so on.
So the total number of ways of arranging r distinct numbers is r(r-1) … 1.

So $\frac{n(n-1)\ ...\ (n-r+1)}{r(r-1)\ ...\ 1}$ gives the number of different r numbers that can be chosen from n numbers in which each specific r numbers is counted only once.

So P(r,n,p) = $\frac{n(n-1)\ ...(n-r+1)}{r(r-1)\ ...\ 1}\ p^r(1-p)^{n-r}$ □

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